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M.Sc. Sem-II, Paper V (Finite Set)

Elementary Set theory, (Finite, Countable and uncountable)

Finite set – With finite number of element a set is called Finite set. A finite set is a set which one could in principle count and finish counting.

$\{2, 4, 6, 8, 10\}$ is a finite set with five elements.

The number of elements of a finite set is a natural number (a non-negative integer) and is called the cardinality of the set.

Finite sets are particularly important in Combinatorics, the mathematical study of counting. Many arguments involving finite sets rely on the pigeon-hole principle, which states that there cannot exist an injective function from a larger finite set to a smaller finite set.

Formally, a set S is called finite if there exists a bijection $f: S \to \{1, ---, n\}$, for some natural n. The number n is the set's cardinality, denoted as $|S|$. The empty set ϕ or $\{\}$ is considered finite, with cardinality zero.

If a set is finite, its element may be written in many ways – in a sequence

$x_1, x_2, ---, x_n \ (x_i \in S, 1 \leq i \leq n)$.

In combinatorics, a finite set with n elements is sometimes called an n-set and a subset with k elements is called a K-subset. For example, the set {5,6,7} is 3-set - a finite set with three elements and {6,7} is a 2-subset of it.

Those familiar with the definition of the natural numbers themselves as conventional in set theory, the so called von Neumann construction, may prefer to use the existence of the bijection

$f: S \to n$, which is equivalent.

Basic properties of Finite Set :- Any proper subset of a finite set S is finite and has fewer elements than S itself. As a consequence, there cannot exist a bijection between finite set S and a proper subset of S. Any set, of this property is called Dedekind-finite. Using the standard ZFC axioms for set theory, every Dedekind-finite set is also finite, but this implication cannot be prove in ZF (Zermelo-Fraenkel axioms without the axiom of choice) alone. The axiom of countable choice, a weak version of the axiom of choice, is sufficient to prove this equivalence.

Any injective function between two finite sets of the same cardinality is also a surjective function (a surjection). Similarly, any surjection between two finite sets of the same cardinality is also an injection.

The union of two finite sets is finite, with

$|S \cup T| \leq |S| + |T|$.

In fact, by the inclusion - exclusion priniciple:-

$|S \cup T| = |S| + |T| - |S \cap T|$.

more generally, the union of any finite number of finite sets is finite. The cartesian product of finite sets is also finite, with: $|S \times T| = |S| \times |T|$.